

Grade 3 Mathematics, Quarter 1, Unit 1.1

Applying Place Value to Addition and Rounding

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Round whole numbers to the nearest 10 or 100.
- Fluently add within 1,000.
- Solve two-step problems using a letter for the unknown quantity, such as problems with result unknown, change unknown, and start unknown ($2 + 3 = ?$, $2 + ? = 5$, $? + 3 = 5$)
- Represent problems using equations with a letter standing for the unknown quantity.
- Determine the reasonableness of answers using mental computation and estimation strategies.
- Identify arithmetic patterns including patterns in the addition table (hundreds chart).

Mathematical practices to be integrated

- Make sense of problems and persevere in solving them.
- Look for entry points to solutions.
 - Identify and execute appropriate strategies to solve a problem.
 - Continually ask, “Does this make sense?”
- Use appropriate tools strategically.
- Use estimations to judge reasonableness of solutions.

Essential questions

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| <ul style="list-style-type: none">• When is it necessary to regroup numbers when adding?• How can you balance an addition equation? (e.g., find the missing number: $22 + a = 44$.)• How can you find a missing addend? | <ul style="list-style-type: none">• What is your strategy for mentally solving addition problems?• What is regrouping?• How can you explain the steps for rounding to the nearest 10 or 100? |
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Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.⁴

⁴ A range of algorithms may be used.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Operations and Algebraic Thinking

3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.³

³ This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Clarifying the Standards

Prior Learning

In grade 2, students were expected to fluently add within 100 using strategies based on place value. They added up to four two-digit numbers. They fluently added within 20 mentally. They used addition within 100 to solve one- and two-step word problems. They skip counted by 5s, 10s, and 100s. They read and wrote numbers to 1,000 using base-ten numerals.

Current Learning

In grade 3, students fluently add within 1,000 using strategies based on place value. Using numbers greater than 100 is at the developmental level. Students use place value to round to the nearest 10 or 100. This is at the developmental level. Students solve two-step word problems using addition. They represent problems using equations with a letter standing for the unknown quantity. This is at the developmental level. They assess the reasonableness of their answers by using estimation strategies. Students identify patterns on an addition chart and hundreds chart.

Routines

- The concept of estimation will become a routine in future units.

Future Learning

In grade 4, students will fluently add multidigit whole numbers using the standard algorithm. They will use place value to round multidigit whole numbers to any place up to one million. They will solve multistep word problems with whole numbers and have whole number answers using the four operations.

Additional Findings

According to *Principles and Standards for School Mathematics*, students should “recognize equivalent representations for the same number and generate them by decomposing and composing numbers” (p. 148).

Principles also states that “students’ development of number sense should continue in grades 3–5, with a focus on multiplication and division. An understanding of the base-ten number system should be extended through continued work with larger numbers as well as with decimals.”

And: “students who understand the structure of numbers and relationships among numbers can work with them flexibly. Students need to have many experiences decomposing and composing numbers in order to solve problems flexibly.” (pp. 148–150)

It also states “research has indicated that beginning with problem situations yields greater problem-solving competence and equal or better computational competence” (p. 68) and “too much developing of procedures brings with it the danger of blind application—even when these procedures are well grounded in experience and reflection (Van Parreren, 1981) (p. 116).

The *K-12 Continuums in Mathematics* located on the RIDE website (<http://www.ride.ri.gov/instruction/curriculum/rhodeisland/expectations/index.aspx>) supports the placement of estimation and mental calculations in the routine section of the scope and sequence. It states that, “students should habitually use estimation when doing mathematics. They should identify when it is appropriate and select a fitting method of estimation. In using estimation, students should determine the level of accuracy needed and analyze the validity of their results.”

This resource further states that “students should habitually make mental calculations involving the addition and subtraction of whole numbers 1 through 20, two-digit multiples of 10, three-digit multiples of hundred, and single-digit numbers with two-digit numbers.” It also states, “during grade 3, students need to be able to solve problems involving the addition and subtraction of whole numbers, including problems that involve regrouping. Particularly important is that students learn to discern *when* to add and *when* to subtract.”

Progressions for the Common Core State Standards in Mathematics (draft) states that: “In grade 3, students continue adding and subtracting within 1,000. They achieve fluency with strategies and algorithms that are based on place value, properties of operations, and/or the relationship between addition and subtraction. Such fluency can serve as preparation for learning standard algorithms in grade 4, if the computational methods used can be connected with those algorithms.”

It also states, “Students use their place value understanding to round numbers to the nearest 10 or 100. They need to understand that when moving to the right across the places in a number the digits represent smaller units.

Grade 3 Mathematics, Quarter 1, Unit 1.2
Using and Applying Addition and Subtraction
Strategies to Solve Problems

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Round whole numbers to the nearest 10 and 100.
- Fluently subtract within 1,000.
- Solve two-step problems using a letter for the unknown quantity.
- Represent problems using equations with a letter standing for the unknown quantity.
- Solve problems with result unknown, change unknown and start unknown, as well as difference unknown.
- Determine the reasonableness of the answers using mental computation and estimation strategies.

Mathematical practices to be integrated

Make sense in problems and persevere in solving them.

- Look for entry points to solutions.
- Identify and execute appropriate strategies to solve the problem.
- Continually ask, “Does this make sense?”

Use appropriate tools strategically.

- Use estimations to judge reasonableness of solutions.

Attend to precision.

- Calculate accurately by rechecking for precision.
- Use clear definitions and state meaning of the symbol they choose including using the equal sign consistently and appropriately.

Essential questions

- When solving a subtraction problem with two- or three-digit numbers, how do you know if you need to regroup?
- How can you find the value that will make an open sentence true or make two expressions equal?
- How can addition be used as a strategy to solve a subtraction problem?
- How are addition and subtraction related?
- What are some ways to show that two expressions are equivalent?
- How can estimation help you determine the reasonableness of your solution?
- How can you balance a subtraction equation (find the missing number: $44 - \underline{\quad} = 22$)?

Written Curriculum

Common Core State Standards for Mathematical Content

Number and Operations in Base Ten

3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.⁴

⁴ A range of algorithms may be used.

3.NBT.1 Use place value understanding to round whole numbers to the nearest 10 or 100.

3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Operations and Algebraic Thinking

3.OA

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

3.OA.8 Solve two-step word problems using the ~~four~~ operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.³

³ This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Common Core Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Clarifying the Standards*Prior Learning*

In grade 2, students fluently subtracted within 100 using strategies based on place value. They subtracted within 1,000 using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction. They understood subtracting three-digit numbers by subtracting hundreds from hundreds, tens from tens, etc. and understood that sometimes they need to decompose numbers to regroup. They subtracted 10 or 100 from a given number 100 to 900 mentally. They explained why subtraction strategies work.

Current Learning

In grade 3, students use place value understanding to round numbers to the nearest 10 or 100. They fluently subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. Fluency is at the developmental level. Students solve two-step word problems using subtraction. They represent problems using equations with a letter standing for the unknown quantity. This is at the developmental level. They assess the reasonableness of their answers by using estimation strategies.

Routines

- The concept of estimation continues as a routine in this unit.

Future Learning

In grade 4, students will fluently add multidigit whole numbers using the standard algorithm. They will use place value to round multidigit whole numbers to any place up to one million. They will solve multistep word problems with whole numbers and having whole number answers using the four operations. They will represent these problems using equations with a letter standing for the unknown quantity.

Additional Findings

Students new to subtraction often apply the commutative property of addition. They are also unsure what equivalency means. According to *A Research Companion to Principles and Standards for School Mathematics*, “many children subtract the smaller from the larger number in each column, to get 26 as the answer to $62 - 48$. This top-from-bottom error is largely eliminated when children learn to subtract with understanding” (p. 71).

Additionally, it states, “research has indicated that beginning with problem situations yields greater problem-solving competence and equal or better computational competence” (p. 68). And, “too much developing of procedures brings with it the danger of blind application—even when these procedures are well grounded in experience and reflection” (Van Parreren, 1981) (p. 116).

According to *Principles and Standards for School Mathematics*, “understanding and using mathematical concepts and procedures is enhanced when students can translate between different representations of the same idea” (p. 139). *Principles* further states, “fluency with whole-number computation depends, in large part, on fluency with basic number combinations—the single-digit addition and multiplication pairs and their counterparts for subtraction and division. Fluency with the basic number combinations develops from well-understood meanings for the four operations and from a focus on thinking strategies” (p. 152).

The *K–12 Continuums in Mathematics* located on the RIDE website (<http://www.ride.ri.gov/instruction/curriculum/rhodeisland/expectations/index.aspx>) states that “during grade 3, students need to be able to solve problems involving the addition and subtraction of whole numbers, including problems that involve regrouping. Particularly important is that students learn to discern *when* to add and *when* to subtract.”

Additionally, the *K–12 Continuums* document supports the placement of estimation and mental calculations in the routine section of the scope and sequence, stating, “students should habitually use estimation when doing mathematics. They should identify when it is appropriate and select a fitting method of estimation. In using estimation, students should determine the level of accuracy needed and analyze the validity of their results.” And, “students should habitually make mental calculations involving the addition and subtraction of whole numbers 1 through 20, two-digit multiples of 10, three digit multiples of hundred, and single-digit numbers with two-digit numbers.”

The *K–12 Continuums in Mathematics* reminds teachers that, when students in grade 3 are working on demonstrating an understanding of equivalency, “the nature of the work with equations does not change substantially from that seen in kindergarten through grade 2. The operations involved can be more complex in the later grades, but the importance of equations in K–3 is to provide the opportunity for students to learn what is meant by the equal sign and to understand properties such as the commutative property of addition.”

Grade 3 Mathematics, Quarter 1, Unit 1.3

Exploring Multiplication

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Recognize that a multiplication fact is the number of groups times the number of objects in each group.
- Solve multiplication problems using strategies.
- Use the commutative property to multiply numbers ($4 \times 6 = 6 \times 4$).
- Identify patterns and explain using properties of operations (including patterns in addition table and multiplication table).
- Use arrays to calculate the area of a rectangle.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Make sense of quantities and relationships in problem situations.
- Use varied representations and approaches in solving problems.

Look for and express regularity in repeated reasoning.

- Look for repeated calculations and patterns.
- Accurate calculations.
- During the process, ask “Does my work make sense?” and at the conclusion, ask, “does my answer make sense?”

Essential questions

- How would you use models, number lines, and explanations to show the relationship between repeated addition and multiplication?
- Explain the strategy you used to solve this problem.
- How can you show that $a \times b = b \times a$?
- How can what you know about multiplication be used to help you solve a problem more efficiently?
- How can patterns on a table help you to find the product in multiplication?
- Explain how an array can help you figure out the area of a rectangle.

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking

3.OA

Represent and solve problems involving multiplication and division.

- 3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

Understand properties of multiplication and the relationship between multiplication and division.

- 3.OA.5 Apply properties of operations as strategies to multiply and divide.² *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) ~~$3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)~~*

² Students need not use formal terms for these properties.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

- 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Measurement and Data

3.MD

Geometric measurement: understand concepts of area and relate area to multiplication and addition.

- 3.MD.7 Relate area to the operations of multiplication and addition.
- Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Common Core Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Clarifying the Standards

Prior Learning

In grade 2, students used addition to find the total number of objects in rectangular arrays with up to 5 rows and 5 columns. They wrote equations to express the total as a sum of equal addends. They skip counted by 2s and used repeated addition to find the total number of objects.

Current Learning

In grade 3, students develop an understanding of the meaning of multiplication of whole numbers through activities and problems involving equal-sized groups, arrays, and area models. They also solve multiplication problems using strategies. They find patterns on addition and multiplication tables. They understand the commutative property for multiplication. This is all at the developmental level.

The routine of estimation should continue in this unit as it applies to the concept of multiplication. Students should use estimation to decide on the reasonableness of a product.

Future Learning

In grade 4, students will interpret a multiplication equation as a comparison. They will represent verbal statements of multiplicative comparisons as multiplication equations. They will use all four operations to solve word problems. They will also find all factors for a whole number from 1–100 and determine whether a whole number is a multiple of a given number.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states that students in the United States go through an experiential progression of multiplication methods that is somewhat similar to that for addition (e.g., Anghileeri, 1989; Baek, 1998; Mulligan & Mitchelmore, 1997; Steffe, 1994). Eventually they make equal groups and count them all (p. 76).

Principles and Standards for School Mathematics states that modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their products represent in various contexts (p. 151).

The book also states, “In grades 3–5, students should focus on the meanings of, and relationship between, multiplication and division. It is important that students understand what each number in a multiplication

or division expression represents. For example, in multiplication, unlike addition, the factors in the problem can refer to different units. If students are solving the problem 24×9 to find out how many legs there are on 29 cats, 29 is the number of cats (number of groups), 4 is the number of legs on each cat (or the number of items in each group), and 116 is the total number of legs on all the cats. Modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their product represent in various contexts” (p. 151).

Elementary and Middle School Mathematics: Teaching Developmentally states, “as with addition and subtraction, there are some multiplicative properties that are useful and thus worthy of attention. The emphasis should be on the ideas and not the terminology or definitions.” It also states, “the array is quite powerful in illustrating the commutative property” (p. 160).

Progressions for the Common Core State Standards in Mathematics (draft) states that: “In equal groups, the roles of the factors differ. One factor is the number of objects in a group, and the other is a multiplier that indicates the number of groups... In the array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. This is useful for seeing commutative property for multiplication in rectangular arrays and areas.”

Grade 3 Mathematics, Quarter 1, Unit 1.4

Developing Multiplication Strategies

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Solve multiplication problems through 100 involving two 1-digit numbers.
- Apply the distributive property of multiplication to find the product of problems.
- Use multiplication to solve word problems involving arrays, equal groups, and measurement quantities.
- Use the area model to represent the distributive property to find the area of a rectangle (i.e., side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$).

Mathematical practices to be integrated

Construct viable arguments and critique the reasoning of others.

- Construct arguments using objects, drawings, diagrams and actions.

Look for and make use of structure.

- Make connections or relationships—patterns
- Hundreds charts/multiplication charts.
- Use manipulatives.
- Use the distributive property.

Essential questions

- Using pictures, words, and numbers, how can you show that multiplying any number by zero results in a product that is always zero?
- When you multiply $1 \times n$, why is the product always n ?
- How can what you know about multiplication be used to help you solve a problem more efficiently?
- Explain what strategies you use to find the product of two factors.
- How does the distributive property help you find the product of two factors?

Written Curriculum

Common Core State Standards for Mathematical Content

Operations and Algebraic Thinking

3.OA

Represent and solve problems involving multiplication and division.

- 3.OA.1 Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*

Understand properties of multiplication and the relationship between multiplication and division.

- 3.OA.5 Apply properties of operations as strategies to multiply and divide.² *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) ~~$3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.)~~ Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)*

² Students need not use formal terms for these properties.

Multiply and divide within 100.

- 3.OA.7 ~~Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.~~

Represent and solve problems involving multiplication and division.

- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.¹

¹ See Glossary, Table 2.

Measurement and Data

3.MD

- 3.MD.7 Relate area to the operations of multiplication and addition.

- c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Common Core Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Clarifying the Standards

Prior Learning

In grade 2, students used addition to find the total number of objects in rectangular arrays with up to five rows and five columns. They wrote equations to express the total as a sum of equal addends. They skip counted by 2s and used repeated addition to find the total number of objects.

Current Learning

In grade 3, students solve multiplication problems with one-digit factors up to the product of 100. Students use the distributive property to decompose a product that they do not know in terms of two products that they know. They also solve problems using arrays, equal groups and measurement quantities. Student use the area model and the distributive property to find the area of rectangles. This is all at the developmental level. By the end of grade 3, students must begin working towards fluency of their multiplication facts.

Routines: Students will continue to use estimation strategies to check for reasonableness of their results.

Future Learning

In grade 4, students will interpret a multiplication equation as a comparison. They will represent verbal statements of multiplicative comparisons as multiplication equations. They will use all four operations to solve word problems. They will also find all factors for a whole number from 1–100 and determine whether a whole number is a multiple of a given number. They will also multiply a whole number of up to four digits by a one-digit whole number, and multiply two 2-digit numbers, using strategies based on place value and the properties of operations. They will continue to use multiplication equations, the area model, and rectangular array.

Additional Findings

A Research Companion to Principles and Standards for School Mathematics states that students in the United States go through an experiential progression of multiplication methods that is somewhat similar to that for addition (e.g., Anghileeri, 1989; Baek, 1998; Mulligan & Mitchelmore, 1997; Steffe, 1994). Eventually they make equal groups and count them all (p. 76).

Principles and Standards for School Mathematics states that modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their products represent in various contexts (p. 151).

The book also states, “In grades 3–5, students should focus on the meanings of, and relationship between, multiplication and division. It is important that students understand what each number in a multiplication or division expression represents. For example, in multiplication, unlike addition, the factors in the problem can refer to different units. If students are solving the problem 24×9 to find out how many legs there are on 29 cats, 29 is the number of cats (number of groups), 4 is the number of legs on each cat (or the number of items in each group), and 116 is the total number of legs on all the cats. Modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their product represent in various contexts” (p. 151).

Elementary and Middle School Mathematics: Teaching Developmentally states, “as with addition and subtraction, there are some multiplicative properties that are useful and thus worthy of attention. The emphasis should be on the ideas and not the terminology or definitions.” It also states, “the array is quite powerful in illustrating the commutative property” (p. 160).

PARCC Model Content Frameworks Mathematics Grades 3-11 states that “Finding single-digit products and related quotients is a required fluency for grade 3. Reaching fluency will take much of the year for many students. These skills and the understandings that support them are crucial; students will rely on them for years to come as they learn to multiply and divide with multi-digit whole numbers and to add, subtract, multiply and divide with fractions. After multiplication and division situations have been established, reasoning about patterns and products can help students remember particular products and quotients. Practice—and if necessary, extra support—should continue all year for those who need it to attain fluency.”